Gravitational collision of small non-spherical particles: swept volumes of prolate and oblate ellipsoids in calm air

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Aggregation of small ice crystals is the main process for formation of precipitating particles in stratiform clouds. Collisions between ice crystals are important for charge separation in anvils of deep convective clouds and lightning formation.

The rate of aggregation of ice crystals depends on the ice crystals shape and on their relative velocity. In this study we present mathematical derivation of gravity induced swept volumes between particles of different shapes and sizes falling due to gravity in calm air. We do not deal here with the problem of collision efficiency. The ice crystals are assumed to be either prolate or oblate spheroids. In contrast to spherical particles, the non-spherical particles can move at some angle relative to the vertical direction of the gravity force. We assume that the particles’ Reynolds numbers are smaller than one (diameter of the order of 100 \( \mu m \)), so that the drag exerted on them by the fluid is Stokesian. We also ignore hydrodynamic interactions between collecting and collected particles, thus assuming that their orientations remain constant within distances while approaching each other.

Mathematical expressions for the relative velocities between non-spherical particles as well as for the geometrical cross-sections are derived. It is shown that the swept volume between ice crystals or between ice crystals and cloud droplets dramatically depends on the aspect ratios of the particles and on their orientations. In many cases the swept volume turns out to be substantially larger, or smaller, than the swept volume of equivalent spherical particles, sometimes by more than an order of magnitude. This result suggest that the simplistic approach of equivalent spheres may lead to serious errors. Due to the nonlinear nature of the collision process such variability of the collision kernel may accelerate the aggregation. These results will be used for calculation of collision kernels in turbulent flows.